Volatility models specifics

This section covers the volatility calculations and interpolation of volatilities for Orc volatility models.

Wing model

Parameters for the Wing model

Column	Description
Expiry	Expiry date.
Days	Days remaining until expiry.
ATM fwd.	The current ATM forward (**F**) value used for the volatility calculation. For an option, a theoretical forward can be calculated (see column picker for more information on theoretical forward). The ATM forward for an expiry date in Volatility manager is the theoretical forward of the options in the expiry date, assuming that the value is the same for all options.
ssr	The skew swimmingness rate (**ssr**) is a skew setting that defines the volatility curve movement along the strike scale with respect to changes in the ATM forward value. It also affects current volatility and current slope calculations. If you set the **ssr** to 100% (completely swimming skew) for the expiry date on the Volatility Manager window, the calculations are based on the synthetic forward price of the contracts for the expiry date using all the relevant parameters of the options. The synthetic is displayed above the skew graph on the Volatility Manager window as the at-the-money (ATM) strike, followed by the ATM volatility in brackets (**[*XX.XX*%]**). This price is referred to as the ATM strike price of the options. For more information about ATM, see "Defining the volatility skew" on page 22-5. To get a well-defined ATM strike for an expiry date, parameters such as base contract, yield curve, and underlying rate need to be the same for all contracts in the expiry. When using **ssr**=0 (fixed skew) for the expiry date, the reference price value is used instead of the ATM strike price. The skew is, therefore, fixed until the reference price (or other skew parameters) change. Hence, you must set the reference price on the Volatility Manager window to use fixed skew. See "Skew parameters table and points" on page 22-9. When using a value for **ssr** between 0 and 100% (partially swimming skew), both the ATM strike price and the reference price are used in the calculation of the central skew point. See "Skew parameters table and points" on page 22-9 for information on how to specify the **ssr** value. For a description of the calculations of actual volatility used for a contract from the volatility surface, see "Wing model" on page 22-36.
Vol. ref	The volatility reference (**vr**) is a skew setting that shows the constant part of the current volatility at central skew point.
VCR	The volatility change rate (**VCR**) is a skew setting that defines dependency of the current volatility at central skew point on changes in the ATM forward value. The volatility change rate (**VCR**) indicates an increase in the volatility when the ATM forward (F) moves down in relation to the reference price (**Ref.price**). Note that **VCR** is multiplied by the **ssr** before being applied. Therefore, the fixed volatility curve (SSR=0) is not affected by the volatility change rate. See "Wing model" on page 22-36. Example: A **VCR** value of 0.2 indicates that the volatility will increase/decrease by 0.2 percentage units per percentage that the ATM is decreased/increased in relation to the reference price (**Ref.price**). The same is true for **SCR**, which in turn affects the slope value in the same way. Note: **VCR** and **SCR** can be excluded when running a portfolio report. You can also choose to omit these values on portfolio windows and trading pages. See "Simulation" on page 24-1.

Column	Description
Curr. vol.	The current volatility (**vc**) at central skew point (Ref is reference price). $vc = vr - VCR * ssr * (F- Ref) / Ref$
Slope ref	The slope reference (**sr**) is a skew setting that shows the constant part of the current slope at central skew point.
SCR	The slope change rate (**SCR**) is a skew setting that defines dependency of the current slope at central skew point on changes in the ATM forward value.
Curr. slope	The current slope (**sc**) at central skew point (Ref is reference price). $sc = sr - SCR * ssr * (F- Ref) / Ref$
Put curv.	The put curvature (**Pc**) is a skew setting that shows the amount of bending of the volatility curve on the put wing between down cutoff and central skew point.
Call curv.	The call curvature (**Cc**) is a skew setting that shows the amount of bending of the volatility curve on the call wing between central skew point and up cutoff.
Down cut off	The down cutoff (**du**) is a skew setting that defines a transition point between the put wing and the down smoothing range. This point corresponds to $X=F*exp(Dcut)$.
Up cut off	The up cutoff (**uc**) is a skew setting that defines a transition point between the call wing and the up smoothing range. This point corresponds to $X=F*exp(Ucut)$.
Down sm.	The down smoothing range (**dsm**) is a skew setting that defines a length of the range on the strike scale where the volatility curve gradually changes from down cutoff to constant volatility level. The length of this range is defined in relation to the length of the put wing. Default value is 0.5.
Up sm.	The up smoothing range (**usm**) is a skew setting that defines a length of the range on the strike scale where the volatility curve gradually changes from up cutoff to constant volatility level. The length of this range is defined in relation to the length of the call wing. Default value is 0.5.
Ref. price	The reference price (**Ref**) is a skew setting that gives a reference value for relative changes in the current forward in order to define the effect **VCR** and **SCR** has on current volatility and slope. In addition, **Ref** is used for the central skew point calculation (**F**) when **ssr** is less than 100%.

Volatility surface

The volatility surface in Orc is defined as a set of volatility skews. Each volatility skew is assigned to a certain expiration date. Depending on the preference setting, these expiration dates can be set as fixed or can shift with time, keeping the interval to expiration date constant for each of the volatility skews. This setting is selected on the Preferences panel and is per client.

Each volatility skew is defined by its skew settings and current forward price (F) for its expiration date..

The skew swimmingness rate (**ssr**) defines the skew movement along the strike scale with respect to changes in the ATM forward price. The 0% **ssr** makes the skew fixed, that is, not sensitive to movements in market prices. When **ssr** is set to 100%, the central skew point follows the ATM forward price. The **ssr** setting can be defined for each expiration date independently.

Volatility skew settings

The volatility skew settings in Orc are a set of the following parameters. The table lists the different parameters, the abbreviations used to refer to them both in the formulas in this chapter (Formula abbreviation) and on the volatility skew settings pane of Volatility Manager (Vol. Man. abbreviation), and the valid value range for each parameter (Valid range).

Volatility skew setting	**Formula abbreviation**	**Vol. Man. abbreviation**	**Valid range**
Reference price	Ref	Ref.price	> 0
Put curvature	pc	Put curv.	
Call curvature	cc	Call curv.	
Down cutoff	dc	Down cutoff	< 0
Up cutoff	uc	Up cutoff	> 0
Volatility change rate	vcr	VCR	
Slope change rate	scr	SCR	
Skew swimmingness rate	ssr	SSR	0%-100%
Down smoothing range	dsm	Down Sm.	> 0
Up smoothing range	usm	Up Sm.	> 0
ATM forward	Atm	ATM /ATM fwd	
Reference (forward) price	Ref	Ref. price	
Derived values			
Current volatility	vc	Curr. vol.	0.05%-400%
Current slope	sc	Curr. slope	

Note The calculated volatility is limited to the same range as the volatility reference parameter.

These parameters together with the "current forward price" (**F**) define the volatility curve for a given expiration date. The volatility curve consists of six parts: two parabolic wings with a beginning at the central point, two smoothing ranges behind each wing, and two constant levels on each side of the skew.

The calculation of the volatility for a given strike can be described as follows.

Firstly, we calculate the current forward price as follows:

- If SSR = 100, F equals the ATM forward.
- If SSR = 0, F equals the reference price for the volatility entry.
- If SSR is somewhere in between, i.e. it is a fraction of swimming skew, then $F = (\mathrm{Atm})^{\frac{ssr}{100}} \cdot (\mathrm{Ref})^{\left(1 - \frac{ssr}{100}\right)}$
  where SSR is any number between 0% and 100% and a zero value corresponds to a fixed skew and 100% corresponds to a swimming skew.

Next, we calculate the current volatility and the current slope at the central skew point. These values may differ from the reference volatility and the reference slope is SSR, SCR and VCR are set.

- Current volatility,
  $$vc = \left(vr - \frac{vcr \cdot ssr \cdot (\mathrm{Atm} - \mathrm{Ref})}{\mathrm{Ref}}\right)$$
- Current slope,
  $$sc = \left(sr - \frac{scr \cdot ssr \cdot (\mathrm{Atm} - \mathrm{Ref})}{\mathrm{Ref}}\right)$$

Thirdly, we introduce an auxiliary variable, the converted strike x. When using the Wing model, the converted strike is $x = \ln(X/F)$. When using the Wing Eurofuture model, it is set to $x = \ln((100 - F)/(100 - X))$ where X is the strike.

Volatility at this converted strike, vol(x) is then calculated as follows:

Between the cut-offs it is essentially a parabola with possibly differing curvatures on each side:

$uc < x \le 0: vol(x) = vc + sc \cdot x + pc \cdot x^2$

$0 < x \le uc: vol(x) = vc + sc \cdot x + cc \cdot x^2$

The smoothing ranges are $dc(1 + dsm) < x \le dc$ and $uc < x \le uc(1 + usm)$, respectively. Here the volatility function interpolates between the before mentioned parabola and a constant value outside the interval $dc(1 + dsm) < x < uc(1 + usm)$.

Explicitly this means:

$dc(1 + dsm) < x \le dc:$

$$vol(x) = vc - (1 + 1/dsm) \cdot pc \cdot dc^2 - \frac{sc \cdot dc}{(2 \cdot dsm)} + (1 + 1/dsm) \cdot (2 \cdot pc \cdot dc + sc) \cdot x - \left(\frac{pc}{dsm} + \frac{sc}{(2 \cdot dc \cdot dsm)}\right) \cdot x^2$$

$dc(1 + dsm) > x:$

$$vol(x) = vc + dc(2 + dsm)(sc/2) + (1 + dsm)pc \cdot dc^2$$

$uc(1 + usm) \ge x > uc:$

$$vol(x) = vc - (1 + 1/usm) \cdot cc \cdot uc^2 - \frac{sc \cdot uc}{(2 \cdot usm)} + (1 + 1/usm) \cdot (2 \cdot cc \cdot uc + sc) \cdot x - \left(\frac{cc}{usm} + \frac{sc}{(2 \cdot uc \cdot usm)}\right) \cdot x^2$$

$uc(1 + usm) < x:$

$$vol(x) = vc + uc(2 + usm)(sc/2) + (1 + usm)cc \cdot uc^2$$

Inter- and extrapolation of the skew settings

The volatility skew settings are interpolated between expiration dates with the assigned volatility skews. Most of the skew settings are interpolated linearly with respect to the time interval until maturity between the closest expiration dates with the assigned skew settings.

For $T_1 < T_X < T_2$

$S(T_X) = (S(T_1) * (T_2 - T_X) + S(T_2) * (T_X - T_1)) / (T_2 - T_1)$

T_X - number of days until expiration date without assigned skew settings

T_1 - number of days until the closest expiration date with assigned skew settings $< T_X$

T_2 - number of days until the closest expiration date with assigned skew settings $> T_X$

S(T) - single skew setting for the given maturity time T

The skew settings assigned to the closest expiration date are used for extrapolation of the volatility surface for expiration dates outside the interval defined in the surface.

Example We have an option with 30 days to expiry, strike price (X) equal to 110 and the theoretical ATM strike (F) equal to 105. Given the set skew parameters below,

Days	Vol	Slope	pc	cc	dc	uc
10	22	-1	2	1	-0.5	0.5
50	21	-0.5	1	0.5	-0.5	0.5

the volatility would be calculated in the following manner:

$$u = (30 - 10)/(50 - 10) = 0{,}5$$

$$vc = 22(1 - u) + 21u = 21{,}5$$

$$sc = (-1) \cdot 0{,}5 + (-0{,}5) \cdot 0{,}5 = -0{,}75$$

$$pc = 2 \cdot 0{,}5 + 1 \cdot 0{,}5 = 1{,}5$$

$$cc = 1 \cdot 0{,}5 + 0{,}5 \cdot 0{,}5 = 0{,}75$$

$$dc = (-0{,}5) \cdot 0{,}5 + (-0{,}5) \cdot 0{,}5 = -0{,}5$$

$$uc = 0{,}5 \cdot 0{,}5 + 0{,}5 \cdot 0{,}5 = 0{,}5$$

$$x = \ln(X/F) = \ln(110/105) \approx 0{,}05$$

Thus the volatility becomes

$$vol(0{,}05) = 21{,}5 + (-0{,}75 \cdot 0{,}05) + (0{,}75 \cdot 0{,}05^2) \approx 21{,}4$$

Time weighted wing model

The time weighted wing differs from the wing model regarding the converted strike x. When using the Wing model, the converted strike is $x = \ln(X/F)$. The transformed strike for the time weighted wing model is $x = \ln(X/F)/\sqrt{(T)}$ where T is the time in years to expiry, floored at one day.

See “Volatility skew settings” on page 22-37 for more information.

Cubic spline (static) model

The Cubic spline (static) model is based on strike prices and implied volatilities and will produce a static surface, i.e. it will not change with the underlying price. This model has one parameter, **Nbr of points** that controls number of spline points.

Fitting

From a users point of view there is no difference between fitting and interpolation in this model. The actual curve fitting in this model only amounts to saving the strike prices and the mean values of all implied volatilities provided to the models fitting algorithm. When fitting is done from the Volatility Manager or from a Trading window, the points that are to be stored on the curve are calculated and a curve is calculated using the cubic spline interpolation.

Interpolation

Interpolation is performed with cubic spline polynomials. Given the strike prices $X_0, X_1, X_2, \ldots, X_N$ and the volatilities $v_0, v_1, v_2, \ldots, v_N$ at these strike prices, the curve is approximated by cubic polynomials on each interval $(X_i, X_{i+1}]$, i.e. for a strike price $X \in (X_i, X_{i+1}]$ the volatility at X is give by

$$P_i(X) = a_i + b_i(X - X_i) + c_i(X - X_i)^2 + d_i(X - X_i)^3$$

The coefficients a_i, b_i, c_i and d_i, $i = 0, 1, 2, \ldots, N-1$, are computed requiring that the resulting polynomials satisfies C^2 regularity at the internal nodes, i.e. that for each $i = 0, 1, 2, \ldots, N-2$

$$P_i(X_{i+1}) = P_{i+1}(X_{i+1})$$
$$P_i'(X_{i+1}) = P'_{i+1}(X_{i+1})$$
$$P''_i(X_{i+1}) = P''_{i+1}(X_{i+1})$$

Moreover, at all nodes X_i, $i = 0, 1, 2, \ldots, N-1$, it holds that $P_i(X_i) = v_i$ and also that $P_{N-1}(X_N) = v_N$. At the endpoints we specify a so-called natural spline condition, $P''_0(X_0) = 0$ and $P''_{N-1}(X_N) = 0$. For a more detailed description of the cubic spline interpolation used in this model, see the Orc Quantitative Guide.